Math 3210

Tutorial 6

Brief Midterm Review

Just one long example:

What is the role of Gaussian elimination and how does it help us with finding optimal solution + how does it link with our previous example???

*. Recall Example from lecture notes:

$$\begin{aligned} x_1 + x_2 - x_3 + x_4 \quad\quad\quad &= 5 \\ 2x_1 - 3x_2 + x_3 \quad + x_5 \quad &= 3 \\ -x_1 + 2x_2 - x_3 \quad\quad + x_6 &= 1 \end{aligned}$$

		x_1	x_2	x_3	x_4	x_5	x_6	
(1)	x_4	1	1	-1	1	0	0	5
(2)	x_5	2	-3	1	0	1	0	3
(3)	x_6	-1	2	-1	0	0	1	1

B

$$B_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

basic solution: $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 3 \\ 1 \end{pmatrix}$

Now what if I want to move from one basic solution to the other?

What if I want to move my basic variable to X_1, X_5, X_6

Change corresponding vector to $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$B_2 = \boxed{\begin{matrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{matrix}}$$

	X_1	X_2	X_3	X_4	X_5	X_6		
X_1	1	1	-1	1	0	0	5	- don't touch.
X_5	0	-5	3	-2	1	0	-7	② - ① ×2
X_6	0	3	-2	1	0	1	6	① + ③

Recall our previous examples + theory:

When we go from one basic solution to the other

let innitially we have

$$\begin{pmatrix} x_{B_1} \\ \vdots \\ x_{B_m} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \Rightarrow \underbrace{\begin{pmatrix} B & \vdots & \end{pmatrix}}_{A} \begin{pmatrix} x_{B_1} \\ \vdots \\ x_{B_m} \\ \vdots \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

dummy

$\cancel{f(x) = c_{B_1} x_{B_1}}$ $f(x) = c_{B_1} x_{B_1} \cdots c_{B_m} x_{B_m}$

$$y = B^{-1} A$$

$$= \begin{pmatrix} I & \vdots & \end{pmatrix}$$

$$= \begin{pmatrix} y_{B_1} & y_{B_2} & \cdots & y_{B_m} & y_{B_{m+1}} & \cdots & y_{B_{m+n}} \end{pmatrix}$$

Recall some methods in the calculation of inverse.

$$\left(\begin{array}{c|ccc} & 1 & 0 & 0 \\ B & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{B^{-1} I}_{B^{-1}} \\ 0 & 0 & 1 & \end{array}\right)$$

Put it back to our example.

$$\left(\begin{array}{c|c} B & \quad \end{array}\right) \Rightarrow \left(\begin{array}{c|c} \underbrace{B^{-1}B}_{I} & B^{-1}(\quad) \end{array}\right) = Y.$$

Now we want to maximised.

$4x_1 + 2x_2 \cancel{+} \overline{-}3x_3 = x_0$ W.R.T.

$x_1 + x_2 - x_3 \le 5$

$2x_1 - 3x_2 + x_3 \le 3$

$-x_1 + 2x_2 - x_3 \le 1$

Under stant

$x_0 = 4x_1 + 2x_2 + 3x_3 + 0x_4 + 0x_5 + 0x_6$

$C = \begin{pmatrix} 4 \\ 2 \\ -3 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_6 \end{pmatrix}$

basic variables

	y_1	y_2	y_3	y_4	y_5	y_6	
	x_1	x_2	x_3	x_4	x_5	x_6	X
x_4	1	1	-1	1	0	0	5
x_5	2	-3	1	0	1	0	3
x_6	-1	2	-1	0	0	1	1
	4	2	-3	0	0	0	0 → x_0

$\underbrace{4 \quad 2 \quad -3 \quad 0 \quad 0 \quad 0}_{c_j - z_j}$

Basic solution $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 3 \\ 1 \end{pmatrix}$ $\quad C = \begin{pmatrix} 4 \\ 2 \\ -3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ → C_B.

Note we chose which basic variable to enter by max ~~z_j~~ - $c_j - z_j$ wher $z_j = y_j \cdot C_B$ i.e x_1 is entering

x_1 is entering.

Then we chose which variable to go by minimising $\boxed{\text{positive}}$ $\frac{x_r}{y_{jr}}$ where x_j is entering

x_5 is leaving.

Want to minimize $\frac{x_r}{y_{1r}}$ $r = 4, 5, 6$

With some elimination, we can get

	x_1	x_2	x_3	x_4	x_5	x_6	X
x_1	1	-1.5	0.5	0	0.5	0	1.5
x_4	0	2.5	-1.5	1	-0.5	0	3.5
x_6	0	0.5	-0.5	0	0.5	1	2.5
		8	1		-2		$\boxed{6}$

$C_j - Z_j$

Red: Basic
Black: Our y

$$C = \begin{pmatrix} \boxed{4} \\ 2 \\ -3 \\ \boxed{0} \\ 0 \\ \boxed{0} \end{pmatrix} \qquad C_B = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$$

$$Z_j = y_j \cdot C_B = y_j \cdot \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$$

x_2 is entering.

$$y_2 = \begin{pmatrix} -1.5 \\ \boxed{2.5} \\ 0.5 \end{pmatrix} \qquad X = \begin{pmatrix} 1.5 \\ \boxed{3.5} \\ 2.5 \end{pmatrix}$$

x_4 is leaving.

x_2 is entering.

$$y_2 = \begin{pmatrix} -1.5 \\ \boxed{2.5} \\ 0.5 \end{pmatrix} \qquad x = \begin{pmatrix} 1.5 \\ \boxed{3.5} \\ 2.5 \end{pmatrix}$$

x_4 is leaving.

	x_1	x_2	x_3	x_4	x_5	x_6	x
x_1	1	0	-0.4	0.6	0.2	0	3.6
x_2	0	1	-0.6	0.4	-0.2	0	1.4
x_6	0	0	-0.2	-0.2	0.6	1	1.8
			-0.2	-3.2	-0.4		

$$C_B = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3.6 \\ 1.4 \\ 0 \\ 0 \\ 0 \\ 1.8 \end{pmatrix}$$ is our optimal solution